Comment on "J and G_c analysis of the tearing of a highly ductile polymer"

Hodgkinson and Williams [1] refer to previously published work by myself and colleagues on the fracture mechanics of highly ductile polymers, including polyethylene, and questions naturally arise concerning the similarities and differences between their work and our own. I am grateful therefore for the opportunity of clarifying a number of points, especially the implication that a "conventional" fracture mechanics analysis is somehow superior to that employed by us.

As the authors correctly imply, the generalized theory of fracture mechanics (GFM) [2] begins by defining a non-linear, apparent, energy release rate $(-\mathrm{d}\mathscr{E}/\mathrm{d}A)$ which is wholly equivalent to $\frac{1}{2}J$ by the Landes and Begley [3] definition. Our definition of the non-linear, inelastic fracture parameter $\mathscr{T}$ (equal to the critical value of $-\mathrm{d}\mathscr{E}/\mathrm{d}A$) pre-dates the Landes and Begley definition, the essence of the generalized theory having been published in 1971 in the context of fatigue crack propagation in polyethylene by Andrews and Walker [4]. GFM differs from the Landes and Begley analysis, however, in that an explicit theoretical formula is derived for $(-\mathrm{d}\mathscr{E}/\mathrm{d}A)$. Thus, for an edge crack specimen under uniform tensile load,

$$-(\mathrm{d}\mathscr{E}/\mathrm{d}A) \equiv \tfrac{1}{2}J = k_1(\epsilon_0)cW_0, \qquad (1)$$

where ϵ_0 and W_0 are the strain and energy input density at points remote from the crack, c is the crack length and k_1 is an explicit function. The critical fracture parameter $\mathscr{T}$ or $\frac{1}{2}J_c$ is thus given directly in terms of the instantaneous crack length and loading conditions (W_0), which represents a significant advance on the use of a purely empirical "apparent energy release rate". Admittedly it has been necessary hitherto to use a compliance method (load–deformation curves on multiple specimens with different crack lengths) to evaluate the function k_1. When this has to be done, the method involves identical experimental work to a Landes and Begley analysis. However, the use of compliance data is not in principle an essential step in GFM since $k_1(\epsilon_0)$ is given explicitly by the theory in terms of the energy density distribution throughout the specimen. It is only our ignorance of this distribution in the general inelastic situation that makes us fall back on compliance tests. Recently some success has been obtained in evaluating the function $k_1(\epsilon_0)$ from first principles using data generated on a single specimen (of ductile steel). This work will be submitted for publication shortly. Rather than being more time consuming than conventional J analyses, therefore, as the authors imply, GFM promises to simplify the determination of $\mathscr{T}$ or J_c on highly non-linear and inelastic materials.

The main thrust of GFM, however, lies in its unequivocal explanation of the physical significance of $\mathscr{T}$ or $\frac{1}{2}J_c$. As long as J_c remains a totally empirical parameter, representing the observed critical value of an admittedly fictional "energy release rate", it cannot be used with any assurance in engineering design, as has been pointed out by others [5]. GFM, on the other hand, gives

$$\tfrac{1}{2}J_c \equiv \mathscr{T} = \mathscr{T}_0 \frac{k_1(\epsilon_0)}{k_1(\epsilon_0) - \sum \beta g \delta v} \qquad (2)$$

or, more simply,

$$\mathscr{T} = \mathscr{T}_0 \Phi, \qquad (3)$$

where $\mathscr{T}_0$ is the energy required to create a unit area of fracture surface in the absence of mechanical energy loss, β is the hysteresis ratio, g is an energy density distribution function and δv is the volume element. The summation is taken over regions of the strain field which unload during crack propagation. Thus, the measured fracture parameter $\mathscr{T}$, is fully specified by the explicit physical and mechanical properties of the continuum. This may appear more significant to a physicist than to an engineer whose only interest may be to characterize a material by a reliable number. It is often, however, engineers who have complained that they do not know what J_c means physically when plasticity supervenes. Furthermore, Equation 2 does offer the unique possibility [2] of predicting and exploring other phenomena such as ductile–brittle transitions, fatigue, notch embrittlement and resistance curve effects and these matters are currently under study.

Under some circumstances it would be correct to identify $\mathscr{T}$ with plane stress fracture and $\mathscr{T}_0$ with plane strain. Then, indeed $\mathscr{T}_0$ would be equivalent to $\frac{1}{2}G_c$ (the elastic energy release rate) as suggested by Hodgkinson and Williams [1].

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Generally speaking, I think this identity applies to elastic–plastic materials where $\mathscr{T}_0$ turns out to be the plane strain craze formation energy, as in polycarbonate and the fibrous fracture of ductile steels [6]. In such materials the transition from plane stress gives rise to a fairly unambiguous ductile-to-brittle transition. However, the parallel breaks down in viscoelastic media, where $\mathscr{T}_0$ excludes all viscoelastic losses while G_c *includes* any such losses as occur in plane stress. Polyethylene is just such a material and it is not therefore surprising that Hodgkinson and Williams obtain a value for G_c which is an order of magnitude greater than the $\mathscr{T}_0$ value of Andrews and Fukahori [7]. A further important factor is that Hodgkinson and Williams assume that polyethylene exhibits a linear elastic retraction curve, which is decidedly not the case as shown clearly by Andrews and Fukahori [7]. It is not therefore generally true, that $\mathscr{T}_0$ is the same quantity as $\frac{1}{2}G_c$; the former has a more precise physical significance as the energy expended at the crack plane in separating the surfaces. In the case where crazing or fibrillation precedes crack propagation, $\mathscr{T}_0$ has to be interpreted in a gross manner as the energy of crazing, but this is only because it is impracticable to treat the craze as a continuous medium and "carry" the general strain field analysis into this microscopical region. In other cases, such as the failure of adhesive interfaces [8] and the tearing of extensible and rubber-like polymers [7], it is possible to identify $\mathscr{T}_0$ directly as a surface energy. This will only be true for $\frac{1}{2}G_c$ if conditions of thermodynamic reversibility are approached. Whilst, therefore, the procedures described in our work [6, 7] for the experimental evaluation of $\mathscr{T}_0$ (as opposed to $\mathscr{T}$) *are* time consuming, they are at present unavoidable. The "simpler" method of Hodgkinson and Williams affords a different (though clearly interesting) quantity. It is perfectly feasible, of course, to computerize the GFM analysis since all it requires is the comparison of deformed and undeformed grids, and some progress has been made in this direction [6].

Finally, a limited amount of work on the GFM analysis of polyethylene film (ICI blown film, code XHF) has been carried out in our laboratory and some data was published in [9]. This shows an initiation value of J_c (or $2\mathscr{T}$) which is independent of crack length for small cracks. This does not necessarily contradict the apparent systematic variation of J_c with crack length (a) shown by Hodgkinson and Williams (Fig. 11 of [1]), but one is inclined to doubt the validity of data where cracks exceed 30% of the specimen width. Our single value of $9.0\,\mathrm{kJ\,m^{-2}}$ for J, agrees well however with their range of 8 to $12\,\mathrm{kJ\,m^{-2}}$ for cracks shorter than 30% of the specimen width. This emphasizes that a difference of a factor of ten between $\frac{1}{2}G_c$ and $\mathscr{T}_0$ is unlikely to arise from variations in the grade of polyethylene used (though Hodgkinson and Williams provide no information about the material employed in their experiments).

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